Communication for maths

Term 2, Week 2: On the formal presentation of curve sketching

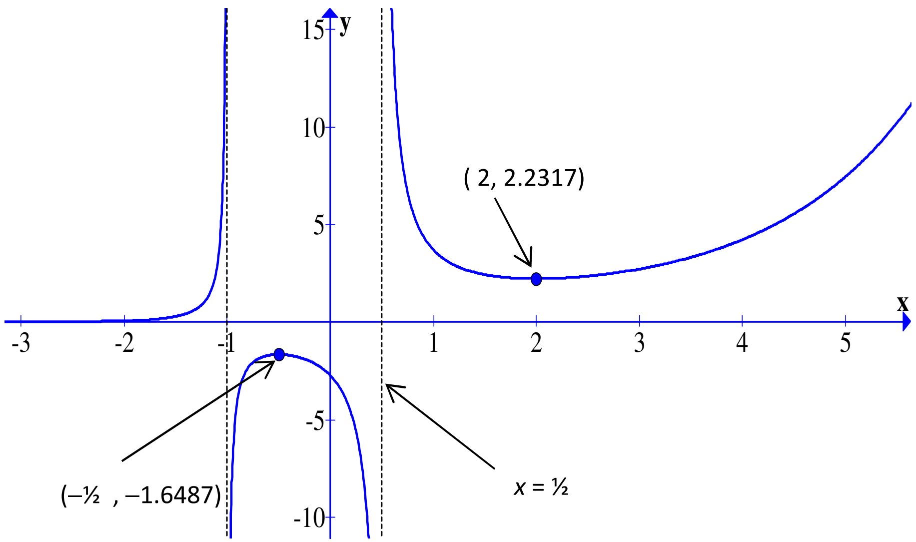
Introduction

- These slides illustrate certain aspects relating to the presentation of curve sketching.
- Some relate to the presentation of all maths in general and some are specific to the presentation of curve sketching.
- Not all aspects of mathematics communication that we have studied so far will be presented here.
- Revise the slides on the previous topics where appropriate.

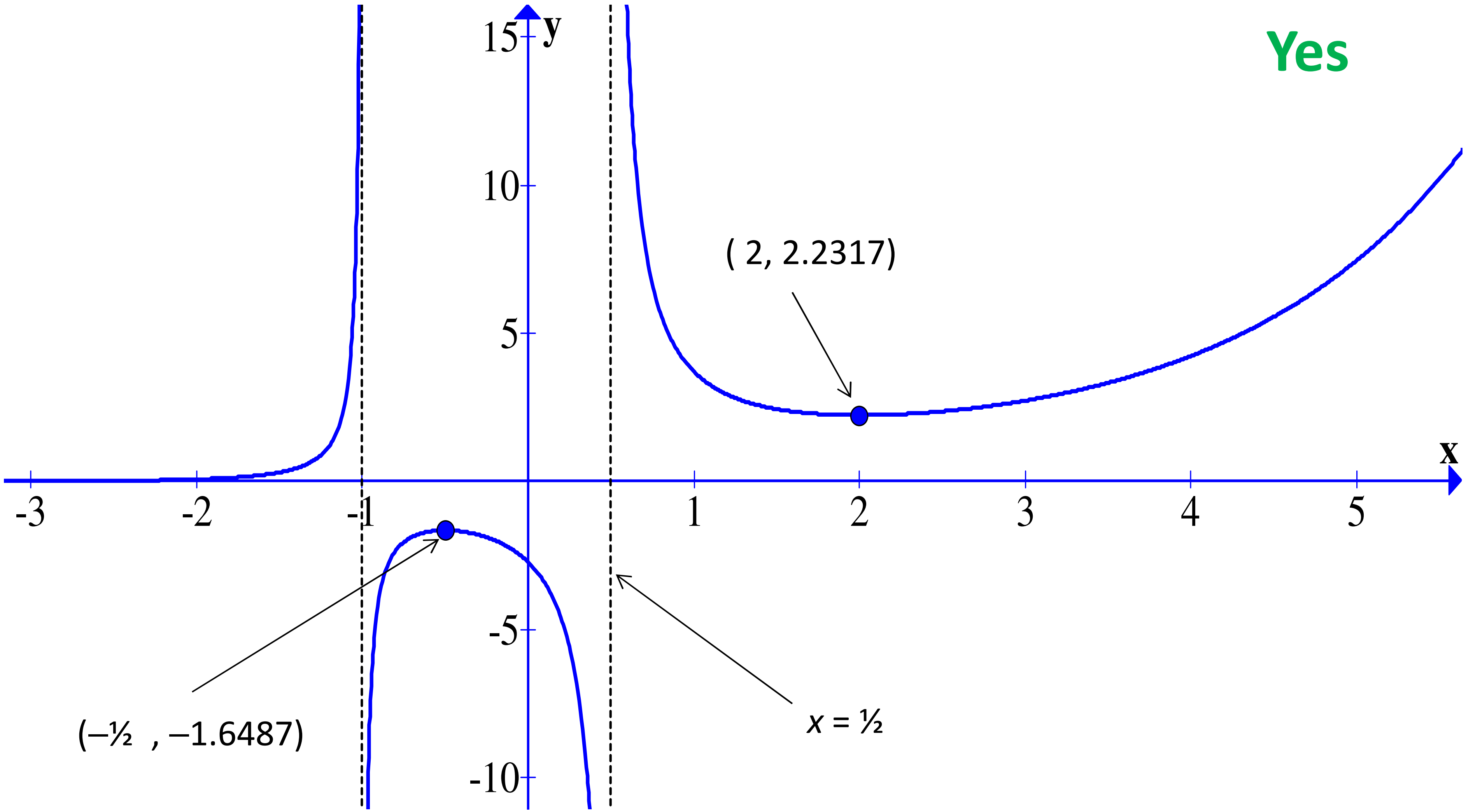
Clean and readable presentation

Your graph should be at least the size of half an A4 page (or it can be bigger)

No



Clean and readable presentation

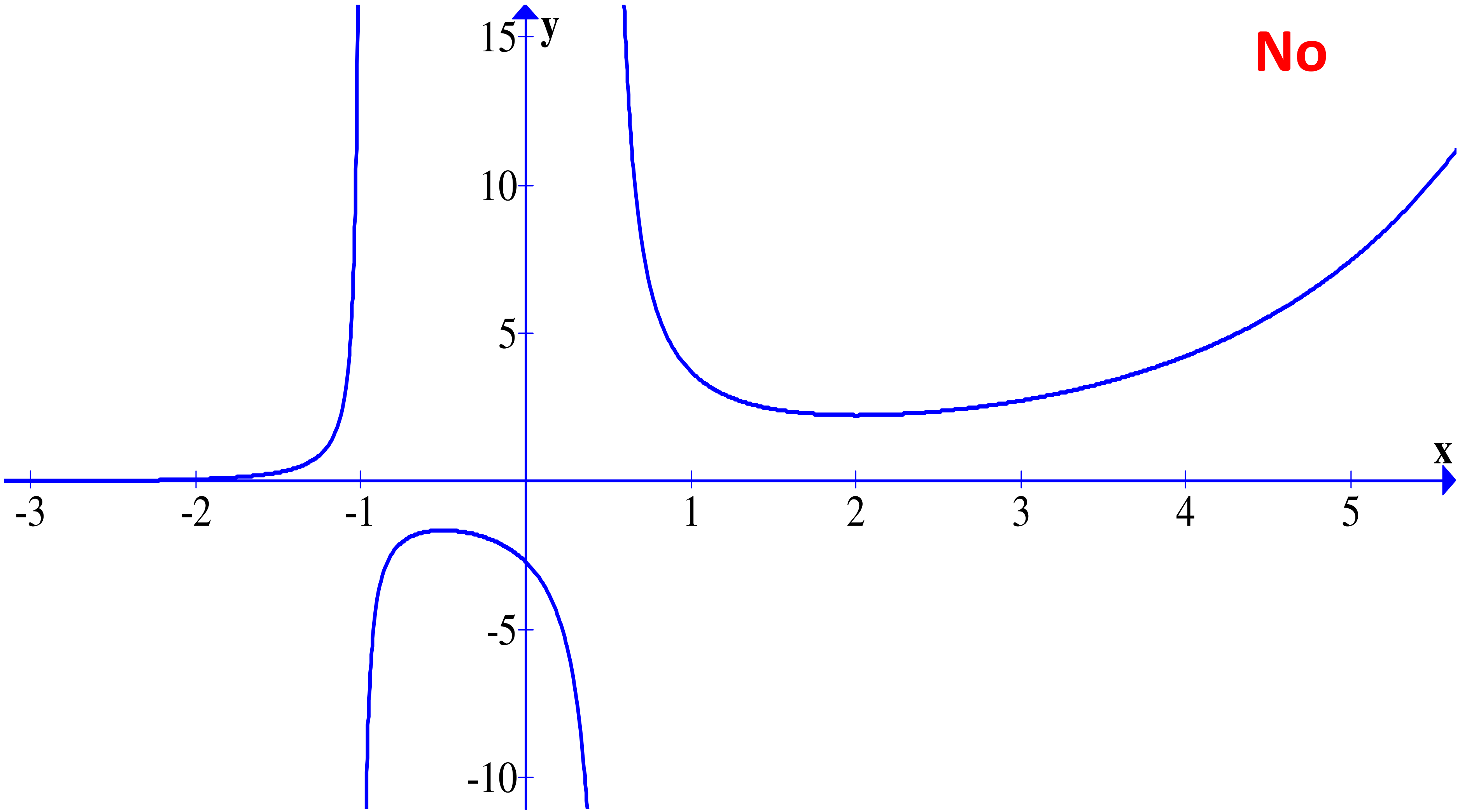


Clean and readable presentation

Your graph should be clearly labelled:

- Show all relevant coordinates;
- Show all asymptotes;
- Show all x and y intercepts;
- Show all stationary points.

Clean and readable presentation



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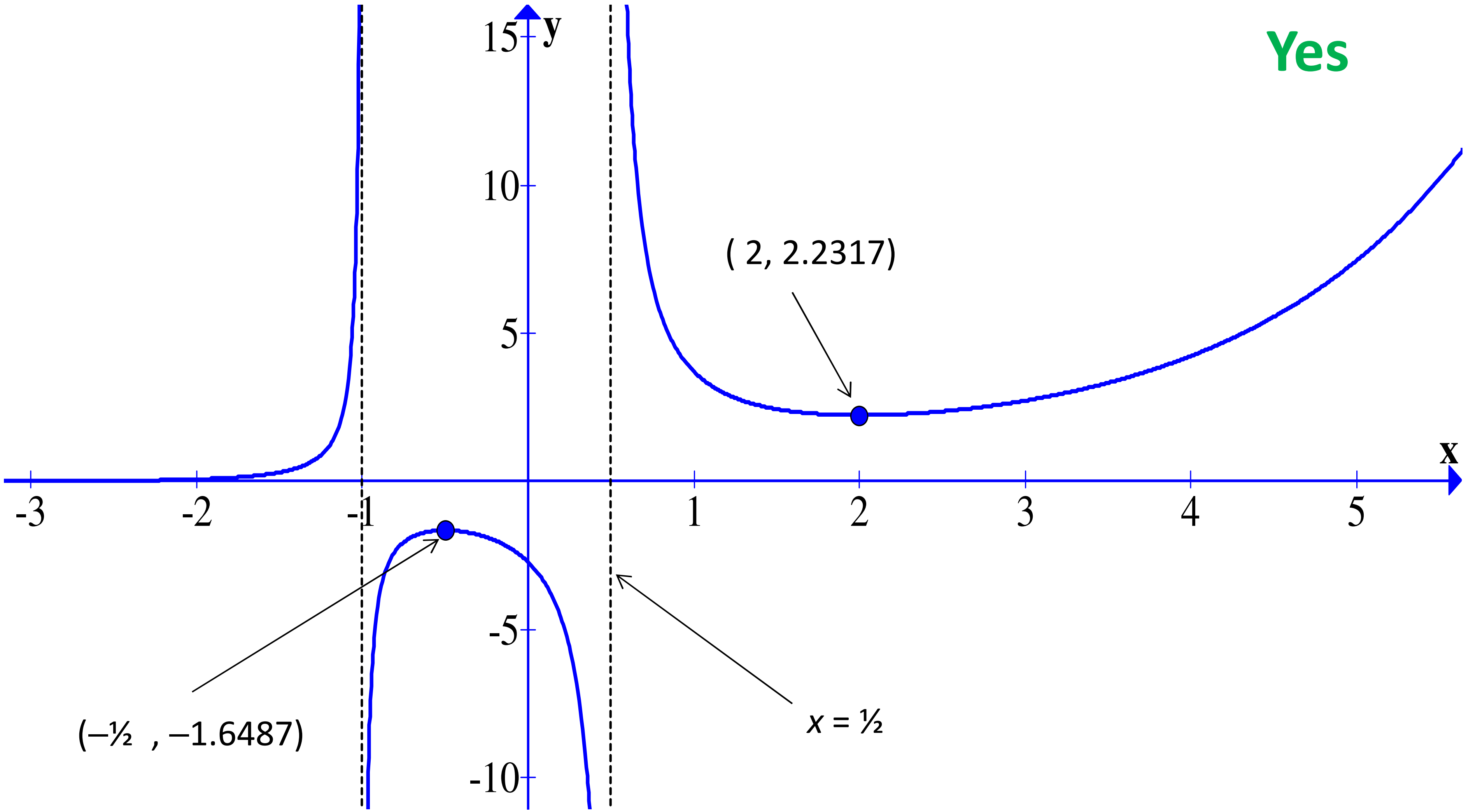
Clean and readable presentation



Justify steps where relevant

- For curve sketching there is a specific type of working to be shown. See your lectures and tutorials for these.

- **To find roots:** For Newton-Raphson method or method of bisection, show formula and each iteration.

Justify steps where relevant

- *Example*

Use the Newton-Raphson method to find the first four iterations of $f(x) = 9x^3 - x - e^x$, starting with the initial value $x_0 = 0.5$

given $x_0 = 0.5$

we have $x_1 \simeq 0.74961$

$x_2 \simeq 0.67287$

$x_3 \simeq 0.66110$

$x_4 \simeq 0.66083$

No

Justify steps where relevant

Yes

Now $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$\therefore \quad x_{n+1} = x_n - \dfrac{9x_n^3 - x_n - e^{x_n}}{27x_n^2 - 1 - e^{x_n}}$

given $x_0 = 0.5$

we have $x_1 \simeq 0.74961$

$x_2 \simeq 0.67287$

$x_3 \simeq 0.66110$

$x_4 \simeq 0.66083$

Justify steps where relevant

- As always read the question to see what is required for a correct solution.

- Also, remember about rounding effects: If the final answers needs to be to 5 d.p. work at least to 6 d.p. (see algebra notes and also comm. In maths slides on binomial theorem for examples on this)

Justify steps where relevant

- **Asymptotes:** For horizontal or vertical asymptotes show all algebra and *arithmetic*
- *Example*

Analyse

$$f(x) = \frac{2x^4 - 3x^3 + 2x + 1}{3x^3 - x^2 + 3}$$

for its end behaviour (i.e what happens to $f(x)$ as $x \to \pm\infty$)

Justify steps where relevant

<u>Solution</u>

$$f(x) = \frac{2x^4 - 3x^3 + 2x + 1}{3x^3 - x^2 + 3}$$

$$= \frac{2x - 3 + 2/x^2 + 1/x^3}{3 - \frac{1}{x} + 3/x^3}$$

So as $x \to +\infty$

$f(x) \to +\infty$

No

Justify steps where relevant

Solution

$$f(x) = \frac{2x^4 - 3x^3 + 2x + 1}{3x^3 - x^2 + 3}$$

$$= \frac{2x - 3 + 2/x^2 + 1/x^3}{3 - \frac{1}{x} + 3/x^3}$$

So as $x \to +\infty$

$$f(x) \to \frac{\infty - 3 + 0 + 0}{3 - 0 + 0} \to \infty$$

Yes

Justify steps where relevant

Present complete steps at each stage.

- *Example*

Stationary Points

given $y = xe^x$, $y' = e^x + x \cdot e^x$

$= e^x(1+x)$

For S.P., $y' = 0 \Rightarrow e^x(1+x) = 0$

No ⟶ so $1+x=0$ ....

Justify steps where relevant

Present complete steps at each stage.

- *Example*

Stationary Points

given $y = xe^x$, $y' = e^x + x \cdot e^x$

$= e^x(1+x)$

For S.P., $y' = 0 \Rightarrow e^x(1+x) = 0$

Yes

$\therefore$ either $e^x = 0$ or $1+x = 0$

But $e^x \neq 0$ so $1+x = 0$ ....

Justify steps where relevant

Justify invalid outcomes.

- *Example*

  This is an incorrect solution. Why?

Question: Sketch $y = \sqrt{1-x} + \sqrt{3+x}$

Solution:

i) Domain: $x \in [-3, 1]$

ii) Intercepts: ....

iii) Stationary points: ....

Sketch: ....

Justify steps where relevant

Justify invalid outcomes.

- *Example*

  Because stage iv) is missing.

iv) Asymptotes :

a) End behaviour

$x \to \pm\infty$ Not a valid consideration since $x \in [-3, 1]$.

b) vertical asymptotes

Function y has no vertical asymptotes because it is not a Rational function or a log function.

Justify steps where relevant

Justify invalid outcomes.

- *Example*

  Justify things that are not valid or irrelevant.

iv) Asymptotes :

a) End behaviour

$x \to \pm\infty$ Not a valid consideration since $x \in [-3, 1]$.

b) vertical asymptotes

Function y has No vertical asymptotes because it is Not a Rational function or a log function.

Justify steps where relevant

Other methods:

- Present complete steps to any use of factor theorem or rational root test (RRT) or numerical method (Newton-Raphson or bisection)
- For the factor theorem or RRT see the algebra slides. For Newton-Raphson see above.

Correct use of symbols

Remember to use the symbol "⇒" correctly

If $f(x) = (x^2-3)\,e^{x+1}$

x intercepts:

Yes $f(x) = 0 \Rightarrow (x^2-3)e^{x+1} = 0$

But $e^{x+1} \neq 0$

Yes $\therefore x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$

y-intercepts:

No $x = 0 \rightarrow f(x) = -3e$

Reminder of some other aspects

No free-standing expressions.

- In mathematics, individual steps/sentences have the symbol “=“, “>”, “<“ (or other).
- We shouldn’t write isolate expressions which show no logical connection to anything.
- Use the relevant logical connective symbol sign to make logical connections.

Reminder of some other aspects

- Align your equal signs.
- Do not write in columns.
- Present clean handwriting.
- No scratch marks or rough work in the presentation of your solution.
- See all previous slides for any other aspect of presentation not mentioned here.

Analysing end-behaviour

What is and isn't okay?

Examples:

1. The following **is okay**

   $f(x) = e^x - e^{-x} \to \infty$ as $x \to \infty$

   $f(-x) = e^{-x} - e^x = -f(x)$

   So $f(x) \to \infty$ as $x \to \infty$.

   And $f(x) \to -\infty$ as $x \to -\infty$.

2. The following **is okay**

   $f(x) = -x^5 + 4x^4 + 3x^2 - 2x + 4$

   As $x \to \infty$, $f(x) \to -\infty$

   As $x \to -\infty$, $f(x) \to \infty$

3. The following is **not okay** because no working is shown.

   $$f(x) = \frac{3x^4 + 2x^2 - 3x - 1}{2x^4 - 3x^3 + x^2 - x - 1}$$

   $f(x) \to \frac{3}{2}$ as $x \to \pm\infty$

Analysing end-behaviour

The following is also **not okay** because step 2 and 3 is not done.

$$f(x) = \frac{3x^4 + 2x^2 - 3x - 1}{2x^4 - 3x^3 + x^2 - x - 1} = \frac{3 + \frac{2}{x^2} - \frac{3}{x^3} - \frac{1}{x^4}}{2 - \frac{3}{x} + \frac{1}{x^2} - \frac{1}{x^3} - \frac{1}{x^4}}$$

$$f(x) \to \frac{3}{2} \text{ as } x \to \pm\infty$$

But the following **is okay**

$$f(x) = \frac{3x^4 + 2x^2 - 3x - 1}{2x^4 - 3x^3 + x^2 - x - 1} = \frac{3 + \frac{2}{x^2} - \frac{3}{x^3} - \frac{1}{x^4}}{2 - \frac{3}{x} + \frac{1}{x^2} - \frac{1}{x^3} - \frac{1}{x^4}}$$

$$f(x) \to \frac{3 + 0 - 0 - 0}{2 - 0 + 0 - 0 - 0} = \frac{3}{2} \text{ as } x \to \pm\infty$$

Analysing end-behaviour

4. The following is **not okay** because when looking at what happens to each term you are left with multiple "∞" symbols to do arithmetic with.

$$f(x) = \frac{3x^2 - 2x - 1}{e^{-x} - 2} \to \frac{\infty - \infty - 1}{0 - 2}$$

The following **is okay**

$$f(x) = \frac{3x^2 - 2x - 1}{e^{-x} - 2} = \frac{3 - \frac{2}{x} - \frac{1}{x^2}}{\frac{e^{-x}}{x^2} - \frac{2}{x^2}} \to \frac{3 - 0 - 0}{0 - 0} = \pm\infty.$$

As $x \to \infty$, $3x^2 - 2x - 1 \to \infty$ and $e^{-x} - 2 \to -2$.

So as $x \to \infty$, $f(x) \to -\infty$.

The following **is also okay**

As $x \to \infty$, $3x^2 - 2x - 1 \to \infty$ and $e^{-x} - 2 \to -2$ so $f(x) \to -\infty$ as $x \to \infty$.

The following **is also okay**

As $x \to \infty$, $3x^2 - 2x - 1 \to \infty$ so $f(x) = \frac{3x^2-2x-1}{e^{-x}-2} \to \frac{\infty}{0-2} = -\infty$ as $x \to \infty$.